# THE UNIQUENESS AND ACCURACY OF POROUS MEDIA MULTIPHASE PROPERTIES ESTIMATED FROM DISPLACEMENT EXPERIMENTS

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**Abstract**

Criteria are presented for validating estimates of relative permeability and capillary pressure functions from laboratory data. Examples are provided to show that methods used to calculate relative permeability from displacement data may provide inaccurate estimates when these criteria are not met.

## Introduction

The relative permeability is a macroscopic property that is defined through extensions of Darcy's law to multiphase flow. These equations can be developed on the basis of local volume averaging[1]. For two-phase flow, the relative permeabilities are taken to be smooth, nonnegative, and nondecreasing functions of the corresponding fluid saturations.

The relative permeability functions can be estimated on the basis of data gathered during displacement experiments. Since the functions are defined within equations, the functions must be estimated from some inverse methodology based on measurements of various quantities which appear within the Darcy and material balance equations that may be used to describe the flow process.

There are a number of different approaches which have been reported for determining estimates of these properties. The quality of the various estimates can vary widely. Selection of the estimation procedures should be made on a sound understanding of the problem at hand. Here, we provide a basic philosophy for estimating relative permeability and capillary pressure functions (collectively called multiphase flow functions), and propose procedures for evaluating candidate estimates. Examples given illustrate the use of the procedures, and also illustrate some problems, which could be encountered in the estimation of the flow functions. The first example illustrates that if the proposed solution criteria are not met by candidate estimates, different solution methods may produce estimates that differ significantly. The second example shows that badly constructed experiments may lead to inverse problems that have inaccurate solutions even if the solution criteria are met.

## The Inverse Problem

The inverse problem is to obtain estimates of the relative permeability (and possibly the capillary pressure) functions on the basis of data measured during displacement experiments. Since the relative permeability is a function of saturation, we require estimates of the entire function. Consequently, for our discussion, any method that calculates only relative permeability *values*, such as the Johnson, Bossler, Naumann (JBN) method[2], should be augmented with an interpolation or smoothing scheme to specify the entire functions (see, e.g.,[17]).

The role of the mathematical model of the displacement experiment in the inverse problem must be understood. All methods for estimating relative permeability are calculational procedures based on a mathematical model of the displacement experiment. Usually, the model is based on some simplified representations of the Darcy and continuity equations for each fluid phase. For example, the JBN method is based on the representation of the sample as being homogeneous, incompressible fluids, no capillary pressure effects, uniform initial (and irreducible) saturation, and constant injection rate for the displacing fluid.

One may say that the inverse problem is inherently ill-conditioned, in that we desire the estimates of (infinite-dimensional) functions on the basis of some finite number of measurements. There is no available theory that guarantees the existence of a single, best solution in such a case. Nevertheless, since we expect the functions to be relatively smooth, we can determine *accurate* estimates of the functions provided there is sufficient information content in the measured data[4].

All estimation procedures deal with finite dimensional representations of the unknown functions. Many methods provide for unique estimates within those finite-dimensional representations. However, injudicious use may result in inaccurate estimates of the functions.

Regardless of the methodology used to generate solutions, the results should be validated. In particular, the following two requirements should be met if a candidate solution is to be retained as estimates of the properties:

1. The mathematical model on which the estimation procedure is based should include all the important physical effects encountered in the experiment.
2. The estimates of the functions should be consistent with the data measured during the experiments.

The second requirement can be evaluated by *predicting* the measured values. It is desired that the predictions are within the accuracy with which the measurements are made. The predictions are made by simulating the experiment using the estimates of the functions with the mathematical model on which the representation of the experiment is based, if all important physical effects have been included (see requirement 1). If any notable physical effects are omitted in that mathematical model, such as capillary pressure, those effects should be included in the simulation of the experiment. Quantitative measures for validating candidate solutions are provided below.

If a valid solution is found, it is to be expected that there may be other functions that also satisfy those criteria. If there is sufficient information content, however, and the candidates are relatively smooth, it is likely that the function values would not differ significantly. Generally, the principle of parsimony[18] could be used to select among candidate solutions. The regression-based method[10] provides for a systematic procedure to choose the simplest representation that satisfies the solution requirements.

## Solution Criteria

The following statistical criteria can be used with the predicted and measured data to evaluate whether a set of estimates of the flow functions is a valid solution to the inverse problem.

Let $Y_i$ be a measured datum from the experiment, and let $F_i(X)$ be the corresponding predicted value given the mathematical model and the estimates $X$. We construct the statistical measure (objective function) $J(X) = [\vec{Y} - \vec{F}(X)]^T W [\vec{Y} - \vec{F}(X)]$, where $W$ is a weigthing matrix chosen as the inverse of the covariance matrix of the measurement errors. If we assume random and uncorrelated measurement errors with normal probability distribution $N(0, \sigma)$, this gives a diagonal weighting matrix with elements $W_i = 1/\sigma_i^2$, and the objective function can be written as $J(X) = \sum_i (\varepsilon_i(X)/\sigma_i)^2$, where $Y_i - F_i(X) = \varepsilon_i(X)$ are the residuals.

For an estimate to fulfill requirement 2 above, the measure $J$ should have a value consistent with our assumptions on the distribution of the measurement errors. For measurement errors with a normal distribution, an approximately normal distribution of the residuals would satisfy this requirement. If the residuals are normally distributed, the sum of squares, $J(X)$, will then have a chi-square distribution $\chi^2$ with $\nu$ degrees of freedoms. The number of degrees of freedom, $\nu$, will depend on the representation used for the estimates. If we have parametrized with $n$ parameters (e.g., $X = X(\vec{\beta})$, $\dim(\vec{\beta}) = n$), $\nu$ will be equal to $m - n$,[16] where $m$ is the number of measurements. From this, $J$ will have an expected value, $E(J)$, of $\nu$, and the standard deviation of the objective function resulting from the random measurement errors will be $\sigma_J = \sqrt{2\nu}$.

We choose as one of the criteria that the value of the objective function of a candidate solution to a given confidence level $\alpha$ should be consistent with a chi-square distribution with an appropriate number of degrees of freedom, $\nu$.

Even if the test on the measure $J$ is passed, we still should check whether the residuals are biased in some way. A statistical measure to do this is the number of runs that the time series of the residuals makes. The number of runs is the number of times the residuals change sign as we move through the data set. It can be defined as:

$$R = \sum_{i=1}^{m-1} r_i, \text{ where } r_i = \begin{cases} 1 & \text{if } \varepsilon_{i+1} \cdot \varepsilon_i < 0 \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

For large $m$, and with the number of positive and negative residuals approximately equal, $R$ will have an approximately normal distribution with an expected value, $E(R)$, of $m/2$, and a standard deviation $\sigma_R = \sqrt{m}/2$, (see e.g.[9], p. 201). If the value of $R$ falls outside an interval with the given confidence level $\alpha$ about $E(R)$, we will have to conclude that the residuals are biased, and that the the predicted values can not be said to be consistent with the measured data.

If the criterion on $J$ is met while the criterion on $R$ is not, this would indicate that our assumption of uncorrelated measurement errors is wrong, or that modeling errors are present. In either case, the estimate should be discarded. Correlated measurement errors could be compensated for by modifying the weighting matrix[19].

This analysis suggests that the following criteria should be met to accept a set of estimates $X$ as a solution of the inverse problem:

1. The value of the objective function, $J(X)$, should pass a hypothesis test on the assumed distribution, i.e.
$$\chi^2_{\nu,\alpha/2} \leq J(X) \leq \chi^2_{\nu,1-\alpha/2}. \tag{2}$$

2. The number of runs, $R(X)$, should pass a hypothesis test on the assumed randomness of $\varepsilon_i$, i.e.,
$$-z_{\alpha/2} \leq \frac{R(X) - m/2}{\sqrt{m}/2} \leq z_{\alpha/2}. \tag{3}$$

## Results and Discussions

We present here some examples to illustrate the use of the solution criteria. A parameter estimation approach is used to estimate candidate solutions, whereby parameters within selected functional forms are determined by minimizing a weighted least squares performance index, taken to be $J$.

### Example 1 – Estimation of $k_{ri}(S_i)$ From Production and Pressure Drop Data

Here we illustrate the estimation of relative permeability from a simulated multirate unsteady state water drainage experiment. The capillary pressure function is fixed, and not part of the estimation problem, and the true data are generated with a simulator[5] using a selected (or "true") set of relative permeability functions. The true functions are constructed by choosing one Corey[6] and one Chierici[7] representation of the relative permeability curves and taking the mean value of these. After constructing the true data sets, "experimental" data are generated by drawing uncorrelated measurement errors from a given normal distribution, $N(0, \sigma)$, and adding them to the true data set.

The course normally followed from here is to choose representations $X = X(\vec{\beta})$, of the flow functions and solve a parameter estimation problem for $\vec{\beta}$. We will demonstrate this solution method with both the Corey and the Chierici representations. The parameter estimation problem can be written as:

$$\min_{\vec{\beta}} J(\vec{\beta}) = [\vec{Y} - \vec{F}(\vec{\beta})]^T W [\vec{Y} - \vec{F}(\vec{\beta})], \tag{4}$$

where the vector of predicted values, $\vec{F}$, now is a function of the adjustable parameters, $\vec{\beta}$.

Utilizing the Corey representation,

$$k_{ri}(S_i) = a_i S_i^{n_i}, i = o, w, \tag{5}$$

a total of four parameters need to be estimated; $\vec{\beta}_{Corey} = [a_w, a_o, n_w, n_o]$. With the Chierici representation,

$$k_{rw}(S_i) = b_w \exp(-d_1 R_g^{d_2}), \quad k_{ro} = b_o \exp(-d_3 R_g^{-d_4}), \quad R_g = \frac{1 - S_w}{S_w - S_{wc}}, \tag{6}$$

six parameters need to be determined; $\vec{\beta}_{Chierici} = [b_w, d_1, d_2, b_o, d_3, d_4]$.

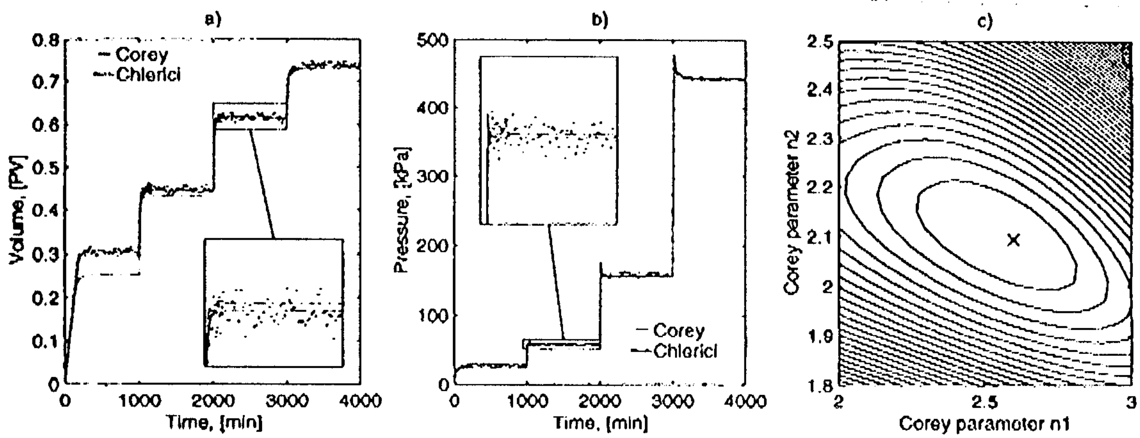


Figure 1: Best match to a) water production and b) differential pressure for the Corey and the Chierici representations, with blow-ups of selected portions of the plot. c) Contour plot of objective function about the minimum (2.6, 2.1) for Corey representation.

We now investigate how well the experimental data can be reconciled using the Corey and Chierici representations. For the same set of experimental data, we estimate $\vec{\beta}_{Corey}$ and $\vec{\beta}_{Chierici}$, solving Eq. 4. Figure 1a) and b) shows the reconciliation of production and pressure drop data, respectively. The figure clearly shows that the Corey representation is not capable of reconciling the experimental data by simulation. Thus, we are not able to find a satisfactory solution to the inverse problem utilizing this correlation. Note, however, that the global minimum of the parameter estimation problem has been achieved. This is illustrated in Figure 1c) where we have plotted the objective function value in a contour plot varying the two Corey exponents, $n_1$ and $n_2$. The two other parameters were kept fixed at their best estimates. The cross in Figure 1c) at $n_1 = 2.6$ and $n_2 = 2.1$ is the solution of Eq.4. A similar plot is obtained if $a_w$ and $a_o$ (or any other combinations) are varied. The Chierici representation leads to a better reconciliation of experimental data, and it is hard to judge whether or not the match is satisfactory based on this plot only.

The estimated relative permeability functions, plotted in Figure 2, are seen to differ noticeably over large saturation regions. Similar results have earlier been interpreted as non-uniqueness of the inverse problem solution[8]. The two functions are both solutions of parameter estimation problems derived from the same inverse problem. This shows that, although the two parameter estimation problems each admits only one solution, these two solutions are not the same.

Using the proposed statistical criteria, we investigate more closely the two estimates. We find that $J(\vec{\beta}^*_{Corey})$ and $J(\vec{\beta}^*_{Chierici})$ are about 20,000 and 1,500, respectively, while the expected value is about 800 with limits of approximately 720 and 870 for a confidence level of 95%. $R(\vec{\beta}^*_{Corey})$ and $R(\vec{\beta}^*_{Chierici})$ are 132 and 316, respectively, with a 95% confidence interval about the expected value of 400 ranging from 370 to 430, approximately. Clearly, both the Corey as well as the Chierici representations are incapable of providing solutions which pass the criteria, and should be discarded.

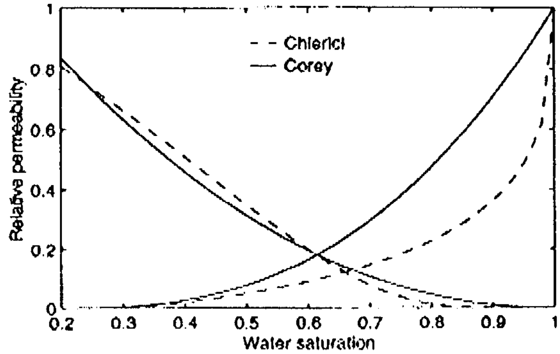


Figure 2: Estimated relative permeability functions in the Corey and Chierici representations.

We now investigate a method which is capable of providing solutions meeting the criteria. It is important to provide the functional representations with sufficient degrees of freedom so that the data are sufficiently predicted. Once achieved, additional degrees of freedom are not necessary since the data do not warrant determination of additional features. This can be accomplished with the regression-based approach[10]. The method has been further developed and tested, and has been reported in a series of articles[3, 10, 11, 13]. Typically, B-splines[12] have been utilized, to represent the functions, although other representations may be equally suitable. First, a representation with just a few basis functions is used to represent each flow function. The corresponding parameter estimation problem is solved. Then, saturation regions where more flexibility is needed are determined[13], and additional basis functions are inserted in such regions. The parameter estimation problem is solved again with this new representation, but using the previous estimated functions as initial values. The method allows for increasing the flexibility in intervals of the curves where it is needed, and thus the use of as few parameters as possible.

This method has been utilized to solve the example problem. We have utilized quadratic B-spline representations of the flow functions, and started the estimation procedure with only 4 parameters for the two curves, adding one parameter for each new step, except for the first, where we added two. In Figure 3, the time series of the residuals are plotted. Figure 3a)-e) shows the residuals using the B-spline representation with increasing numbers of parameters, while Figure 3f) and g) show the results for the Chierici and the Corey representations, respectively. It can be observed that the pattern in the residuals for the first B-spline estimate is quite close to that of the Corey representation. The residual objective function and run values are shown in Figure 4. Again the Corey and first B-spline estimates (denoted "step 1" and "step 2" in the figure) are quite close, while the Chierici values are better. The expected values of $J(\vec{\beta}^*)$ and $R(\vec{\beta}^*)$ with one and two standard deviations are shown as ellipticly shaped regions in the figures. As the number of parameters in the B-spline representation is increased, it is observed that the values of $J(\vec{\beta}^*)$ and $R(\vec{\beta}^*)$ get closer to the expected values, and, finally, the obtained values are within two standard deviations (the whole elliptical contour) of the expected values (step 8). The estimated curves at step 8 have the fewest number of parameters with sufficient flexibility to satisfy the statistical criteria, and is chosen as our solution. A further increase in the number of parameters may lead to

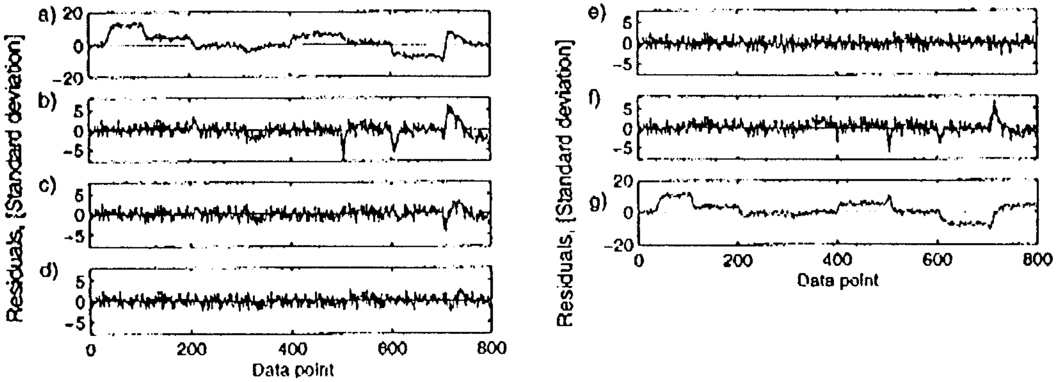


Figure 3: Time series of residuals for water produced (points 1-400) and for differential pressure (points 401-800). a)-e) Steps 2, 3, 5, 7 and 9 of spline expansion, with 6,7,9,11 and 13 estimated parameters, respectively, f) Chicrici and g) Corey representations, with 4 and 6 estimated parameters.

an improved match, but would also lead to flexibility in excess of what is needed. As the variance error[4] increases with increased flexibility, giving less accurate parameter estimates, we stop the regression-based method at the first estimate that meets the statistical criteria.

In the *solution criteria* section the probability distributions of the statistical measures were discussed. These distributions were the result of normally distributed measurement errors. To illustrate the distribution, four other realizations of the measurement errors were used to construct four other estimation problems, and the resulting values of the statistical measures are plotted as "alt.". We note that the spread in the values is of the order of one standard deviation (the dashed elliptical contour).

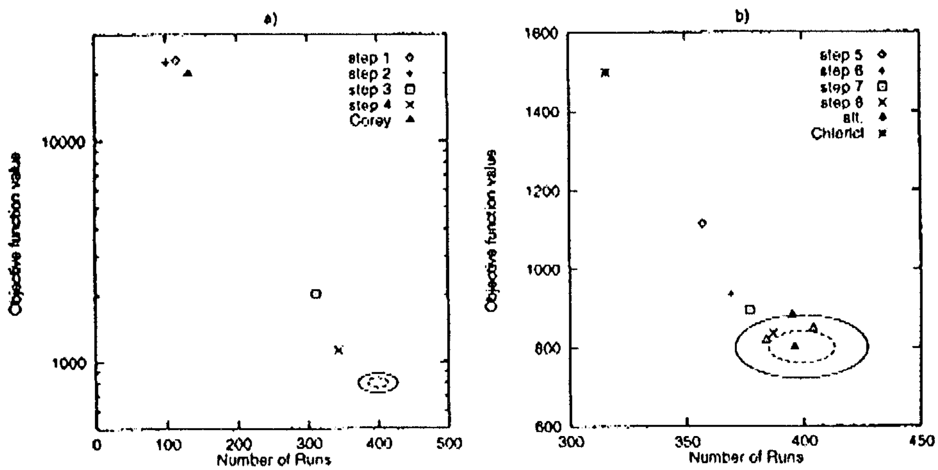


Figure 4: Number of runs vs. objective function value. a) Step 1-4 of spline expansion and the Corey representation. b) Step 5-8 of the spline expansion and the Chicrici representation.

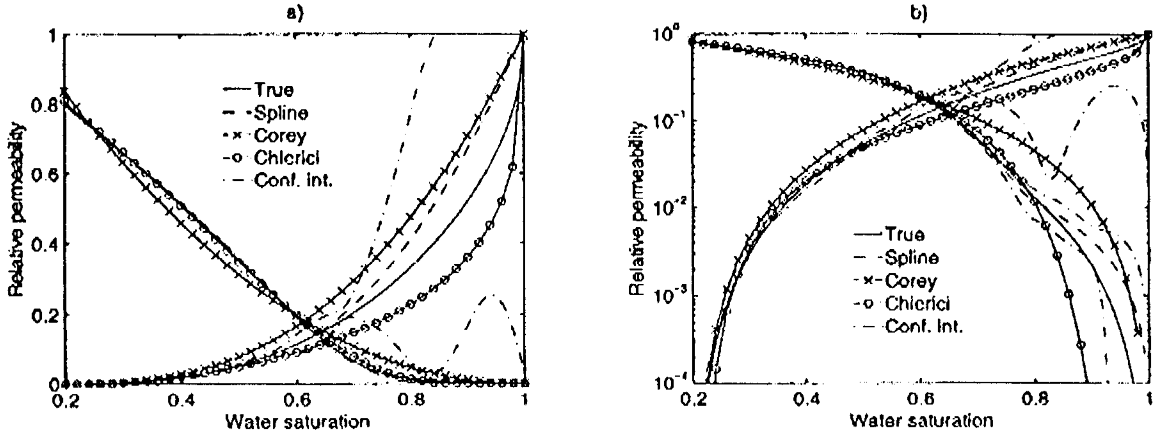


Figure 5: a) Linear and b) logarithmic plot of the estimated spline relative permeability curves of step 8, together with confidence intervals and the true curves. The Corey and Chierici curves from Fig. 2 are plotted for comparison.

Once a candidate solution is found that meets the statistical criteria, the accuracy of the estimates can be assessed. To get a quantitative measure of the accuracy of the estimates, we investigate the sensitivity of the objective function near the obtained minimum. If $F(\vec{\beta}^*)$ is a set of predicted data satisfying the criteria above, and if $F$ is a smoothly varying function of $\vec{\beta}$, all parameters within a region $S$ of parameter space around the minimum $\vec{\beta}^*$ will give values of $J$ and $R$ acceptable by the criteria. The size of $S$ is directly related to the accuracy of the parameter estimates. One method for calculating the size of $S$ is the linearized covariance analysis[4]. This analysis provides approximate confidence intervals around the estimated flow functions, and gives a quantitative measure of the region of acceptable solutions. This analysis has been utilized to determine the accuracy of flow function estimates[4] and to design experiments[15].

We have utilized the linearized covariance analysis to determine the confidence intervals of the estimated flow functions in Example 1. The results from this analysis are shown in Figure 5. The flow functions are well estimated in the lower water saturation region, while for higher saturations the confidence interval for $k_{rw}$ become large. In that region the flow functions are poorly determined. We see that the true solution lies within the confidence band of the last step in the regression-based approach. Note that both the Chierici and the Corey functions are outside of this band for most of the saturation region, and thus provide inferior estimates.

As illustrated above, accurate, although not necessarily unique, estimates may be obtained provided that a flexible representation of the flow functions is utilized and that there is sufficient information in the data. As long as the estimate is accurate, the non-uniqueness will not destroy the value of the estimate, as the function values of those solutions meeting the criteria will are expected to be close. An estimate of *how* close is given by the confidence interval.

Individual solution candidates may still be significantly different, as in the case of the Chierici and Corey representation above. The fact that they differ is not a question of non-uniqueness, simply because the term non-uniqueness only is used for *solutions* of the inverse problem; the Corey and Chierici representations are incapable of providing solutions to the inverse problem in Example 1.

There are situations for which the information content in the data is not sufficient to determine the functions. This is shown in the next example.

**Example 2 – Estimation of $k_{ri}(S_i)$ From Production Data**

Consider estimation of the relative permeability curves from a constant rate experiment where the capillary forces are negligible and only production data are available.

The process is adequately described by the Buckley-Leverett equations[14]. This means that the saturation of the phases (and hence, the production data) are determined by the fractional flow function, $f = 1/(1 + (k_{ro}\mu_w)/(k_{rw}\mu_o))$, only. The fractional flow is dependent upon the saturation only through the ratio $k_{ro}/k_{rw}$. Consequently, a solution of the estimation problem, satisfying the statistical criteria, can be achieved by any two flow functions $k_{ro}$ and $k_{rw}$ with this ratio.

We conclude that the the information content in the production data is not sufficient to determine the individual relative permeability functions. More production data or more accurate data (smaller standard deviation of the error distribution) would not improve the situation in this case. This is a case of an inverse problem with a truly non-unique solution, and we believe the term non-unique should be reserved for this kind of problems.

## Conclusions

1. Criteria for validating estimates of relative permeability and capillary pressure functions from laboratory data have been presented.

2. Examples have been provided to show that methods used to calculate relative permeability from displacement data may provide inaccurate estimates when these criteria are not met.

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