

# A NEW VERSATILE RELATIVE PERMEABILITY CORRELATION

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## ABSTRACT

There are at least two key aspects of simulating multi-phase flow experiments. One is the actual estimation of multiphase flow properties from measured data, and the other is the representation of the unknown functions for relative permeability and capillary pressure. It is essential that the representation of these functions have sufficient degrees of freedom to model the measured data whilst remaining straightforward and easy to communicate.

A new smooth and flexible 3 parameter analytical correlation for relative permeability is proposed as a possible replacement for currently adopted industry standards. Results from e.g. steady-state relative permeability experiments often exhibit behaviour which is difficult to model using e.g. the Corey correlation. The new correlation influences different parts of the relative permeability curve and thereby captures variable behaviour across the entire saturation range.

The strength of the new correlation is demonstrated by utilizing steady-state experiments performed at reservoir conditions on core samples from the Norwegian Continental Shelf. The new correlation has been included in a core flow simulator, and the experimental data has been reconciled through an automated optimisation routine. The benefits of applying this correlation during full-field simulation are also demonstrated.

## INTRODUCTION

Relative permeability is routinely measured for input to full field reservoir models. Experiments are frequently performed by injecting either water or gas into an oil-filled sample at initial water saturation. Analysis of these experiments, either analytical or numerical, gives relative permeability information located towards the residual oil saturation, often covering only 5-15 percent of the entire saturation range. With such a limited saturation range the use of a correlation like Corey [1] is relatively straightforward. This is a simple power law function with only one empirical parameter, the power itself. Values of residual oil saturation,  $S_{or}$ , initial water saturation,  $S_{wi}$ , etc. are generally not regarded as adjustable parameters. However, to achieve an acceptable model for field applications, experimentation must be performed to capture the entire saturation range, from high oil saturation,  $S_{wi}$ , toward low oil saturation,  $S_{or}$  [2, 3]. The Corey model and similar models frequently show limitations to exhibit the flexibility that is required to represent relative permeability for the entire saturation range.

A simple improvement to the Corey model was suggested by Sigmund & McCaffery [4], by adding a linear term with an empirical coefficient to the standard power term in the Corey correlation. Chierici [5] proposed a 2-parameter correlation based on the exponential function. This correlation is more flexible than the previously mentioned. However, it may not be adequate as each of the parameters influences the curve in the entire saturation range. More flexible functions like B-splines have been proposed and honoured through several papers; see [6] and its references. However, B-splines and spline-derivatives may create one or several breaks in the relative permeability curves [7, 8], and they can also be difficult to communicate for field applications due to the numerous parameters.

The authors acknowledge that numerous alternative correlations are currently in use; however, the new correlation is discussed and compared alongside the most common of the published industry standard correlations. A frequent characteristic of the published correlations seems to be that none of them may be capable to describe the relative permeability curves in the entire saturation range, i.e. for low, intermediate and high water saturations. To adequately model the relative permeability as a smooth, although flexible curve in the entire saturation range, a new 3-parameter analytical correlation have been developed and included in the commercial core flow simulator Sendra [9] for interpretation purposes.

## NEW CORRELATION - LET

A new versatile 3-parameter correlation is proposed in order to gain flexibility and a proper curve over a wide saturation range. The correlation should be able to show similar behaviour at both low and high oil saturation. A practical way to transfer this into a mathematical formula is to use both oil saturation and water saturation in the correlation. Further, it is practical that the mathematical elements in the correlation are finite and that values of one and zero are achieved as values, not as limits. A rational function which is extended to include arbitrary powers will fulfil the above requirements.

The proposed correlation is described by 3 parameters  $L$ ,  $E$ ,  $T$ . For a water-oil flow, the parameters for oil relative permeability are written as  $L_o^w, E_o^w, T_o^w$  where the subscript denotes the oil phase and the superscript denotes the water phase. The correlation for oil and water relative permeability with water injection is thus

$$k_{row} = k_{ro}^x \frac{(1 - S_{wn})^{L_o^w}}{(1 - S_{wn})^{L_o^w} + E_o^w S_{wn}^{T_o^w}} \quad (1)$$

$$k_{rw} = k_{rw}^o \frac{S_{wn}^{L_w^o}}{S_{wn}^{L_w^o} + E_w^o (1 - S_{wn})^{T_w^o}} \quad (2)$$

where the normalized water saturation is

$$S_{wn} = \frac{S_w - S_{wi}}{1 - S_{wi} - S_{orw}} \quad (3)$$

Only  $S_{wi}$ ,  $S_{or}$ ,  $k_{rw}^o$  and  $k_{ro}^x$  have physical meaning, while the parameters  $L$ ,  $E$  and  $T$  are empirical. The parameter  $L$  describes the lower part of the curve, and by similarity and experience the  $L$ -values are comparable to the appropriate Corey parameter. The parameter  $T$  describes the upper part (or the top part) of the curve in a similar way that the  $L$ -parameter describes the lower part of the curve. The parameter  $E$  describes the position of the slope (or the elevation) of the curve. A value of one is a neutral value, and the position of the slope is governed by the  $L$ - and  $T$ -parameters. Increasing the value of the  $E$ -parameter pushes the slope towards the high end of the curve. Decreasing the value of the  $E$ -parameter pushes the slope towards the lower end of the curve. Experience using the LET correlation indicates that the parameter  $L \geq 1$ ,  $E > 0$  and  $T \geq 0.5$ .

The LET correlation is developed for use in special core analysis and reservoir simulation in general. Further, the correlation will show its merits for oil and water in the following paragraphs, and the formulas are repeated in Appendix A for clarity and completeness. The endpoint value of relative permeability may be interpreted as a non-normalized relative permeability which is a function of non-normalized saturations with values at the initial or residual saturations. The LET formula is also able to correlate endpoint values for both oil and water relative permeability using non-normalized saturations (see Appendix A), and this capability follows from its flexibility and ability to handle s-behaviour of the normalized relative permeability. The s-behaviour has earlier been discussed for gas-oil flow [10], and for water-oil flow [11]. The LET correlation is also extended to cover gas-oil flow (Appendix B) and water-gas flow (Appendix C), and its capabilities to handle these systems follows from its flexibility and its merits to handle s-behaviour for water-oil flow.

## INTERPRETATION OF SCAL EXPERIMENTS

To test and demonstrate the strength of the new correlation, interpretation of two steady-state type experiments are shown: one that requires the *s-behaviour* of the oil relative permeability while the other requires the *flexibility* that is the nature of the new correlation. Interpretation of relative permeability and reconciliation of the experimental data (i.e. differential pressure and production) have been performed by utilizing the optimization routine in the commercial core flow simulator Sendra [9]. None of the experimental cases can be satisfactorily reconciled by the conventional industry standards like Corey [1], Sigmund & McCaffery [4] or the Chierici [5] correlation. The Sigmund & McCaffery correlation behaved almost identical as the Corey correlation, hence the results are not shown in the below figures.

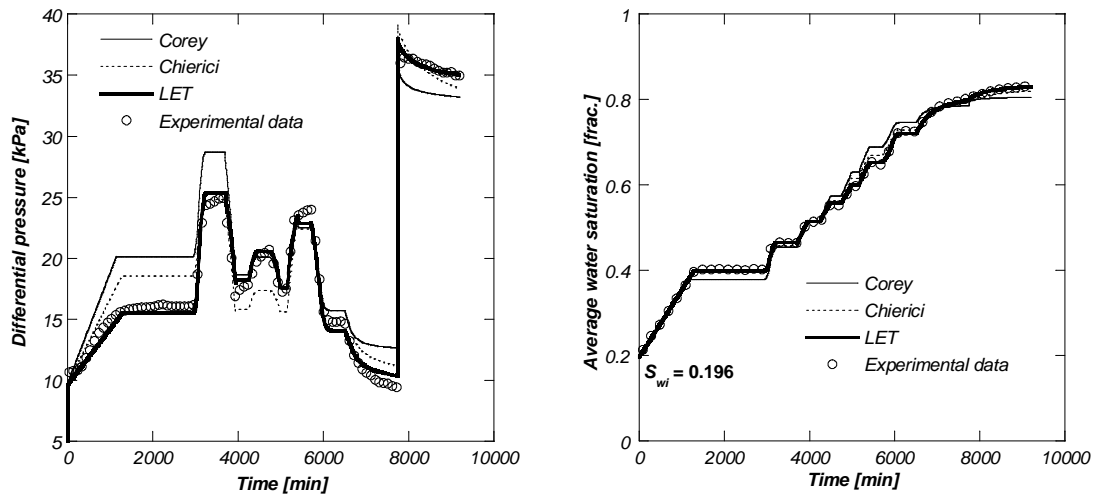
Both cases are performed on a composite of reservoir rock material from the Norwegian Continental Shelf performed horizontally at reservoir conditions. The rates are kept high to avoid severe capillary pressure effects. Core and fluid properties are given in table 1 together with the experimental conditions. Both use live reservoir oil and synthetic formation water at equilibrium.

**Table 1:** Core and fluid properties for the SCAL experiments.

Property	Case no.1	Case no.2
Length, core [cm]	31.13	11.74
Diameter, core [cm]	3.74	3.77
$k_o(S_{wi})$ [mD]	2396	1042
$S_{wi}$ [frac.]	0.196	0.079
Porosity [frac.]	0.296	0.27
Viscosity, water [cP]	0.395	0.306
Viscosity, oil [cP]	2.41	0.67
Temperature [ $^{\circ}$ C]	84	98
Pore pressure[kPa]	25 000	38 900
Flow rates [ml/min]	Oil: 1.984; 1.88; 0.78; 0.5; 0.22; 0.12; 0.016; 0.0; 0.0 Water: 0.016; 0.12; 0.22; 0.5; 0.78; 1.88; 1.984; 2.0; 8.0	Oil: 4.96; 2.5; 0.3; 0.04; 0.0; 0.0 Water: 0.04; 2.5; 4.7; 4.96; 5.0; 8.0

### Case 1 – S-behaviour of Oil Relative Permeability

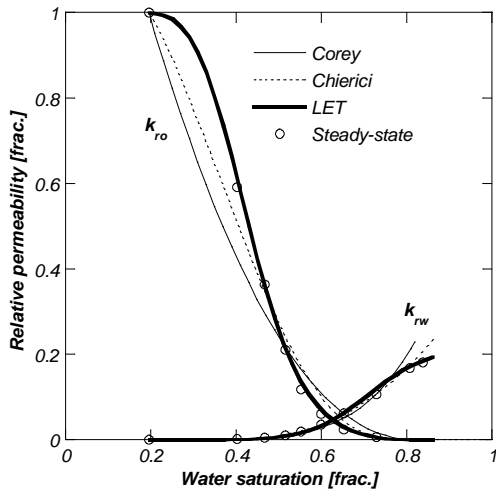
Figures 1 and 2 show the experimental data, the history match when the industry standard correlations [1, 5] are utilized and the history match when the new correlation is used in the optimization procedure.



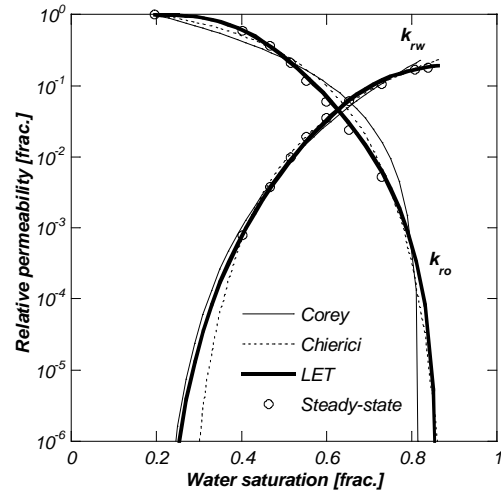
**Figure 1:** History match of differential pressure **Figure 2:** History match of production

None of the conventional correlations are flexible enough to reconcile the entire set of experimental observations. The reason for this is that these correlations are described with only one or two parameters and thus suffer by bias error [12]. This is most pronounced for differential pressure, figure 1, which is the experimental data that contains most information of the magnitude of the relative permeability. However, the production data, figure 2, also suffer for the limited flexibility.

The new correlation exhibits sufficient flexibility to satisfactorily reconcile the entire set of experimental data, both differential pressure and production until the experiment is accomplished. Even though the new correlation is flexible, it maintains a smooth behaviour as shown in figure 3 and 4.



**Figure 3:** Relative permeability; lin-lin.



**Figure 4:** Relative permeability; semilog.

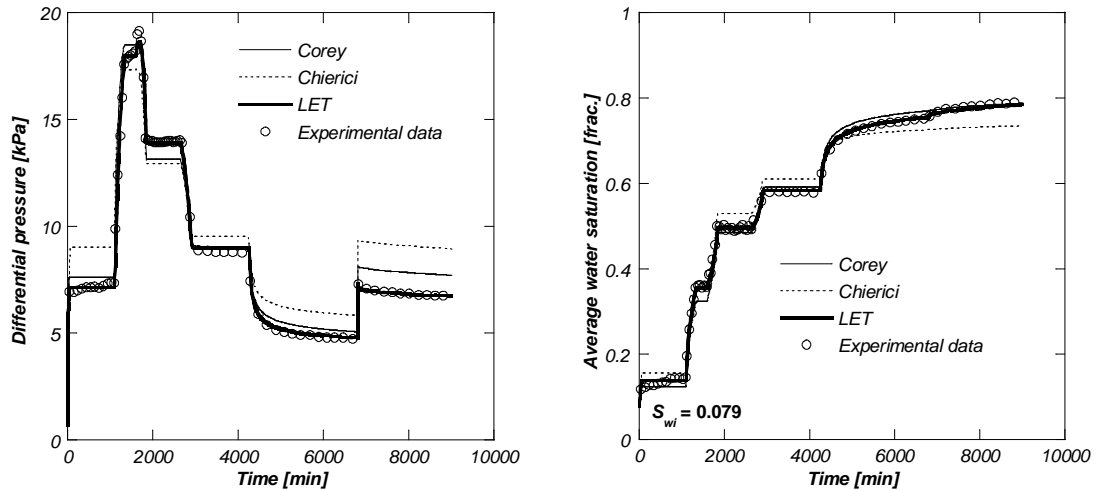
While the water relative permeability is almost equal for all correlations, the oil relative permeability exhibits s-behaviour at low water saturations. This s-behaviour is very well modelled by the new correlation. Due to the high rates and long core sample, the capillary pressure effect is minimized and the new correlation follows the analytical steady-state points. The conventional correlations are neither capable to retain the s-behaviour for oil relative permeability at low water saturation nor to mimic the analytical steady-state points.

The s-behaviour of the oil relative permeability is occasionally observed for flow processes whenever the experiment is adequately performed to identify that behaviour, i.e. steady-state type experiments. A possible explanation of the s-behaviour is suggested by honouring the wettability of the core sample. Wettability measurements of neighbour core samples at comparable conditions indicate the wettability to be mixed-wet to weakly water-wet. Water is thus represented in the very smallest pores, along the pore wall and corners of medium sized pores. Oil will be present in the middle part of medium sized pores and probably entirely in the larger pores. When water enters the core sample, it will be imbibed into the water-wet small/medium sized pores where oil and water is present, before water is flooded in the larger pores. The process will initially be a spontaneous imbibition process into the smallest/medium sized pores rather than a flow process into the larger pores. The small pores do not contribute significantly to the oil permeability, and the negative slope of the oil relative permeability curve will thus be small at low water saturations. When water saturation increases, the slope turns steeper as water enters the larger pores which

contribute to lowering the oil permeability. The wettability, pore shape and pore-size distribution affect the relative permeability in general, and the s-behaviour is probably a complex interaction of these properties.

### Case 2 – Flexible Behaviour of Relative Permeability

Figures 5 and 6 show the experimental data, the history match when the industry standard correlations [1, 5] are utilized and the history match when the new correlation is used in the optimization procedure.



**Figure 5:** History match of differential pressure **Figure 6:** History match of production

None of the conventional correlations are flexible enough to satisfactorily reconcile the experimental observations in the entire saturation range. As for case 1, the reason for this is that these correlations are described with only one or two parameters and thus suffer by bias error [12].

The new correlation exhibits sufficient flexibility to satisfactorily reconcile the entire set of experimental data, both differential pressure and production until the experiment is accomplished. Even though the new correlation is flexible, it maintains a smooth behaviour as shown in figure 7 and 8. The flexibility of the new correlation is required for giving both water- and oil relative permeability a proper shape in the entire saturation range.

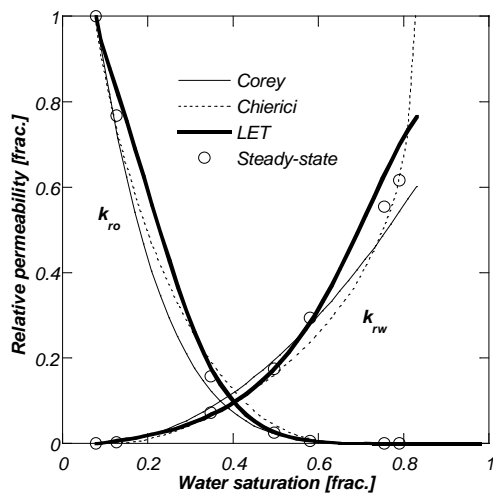


Figure 7: Relative permeability; linlin.

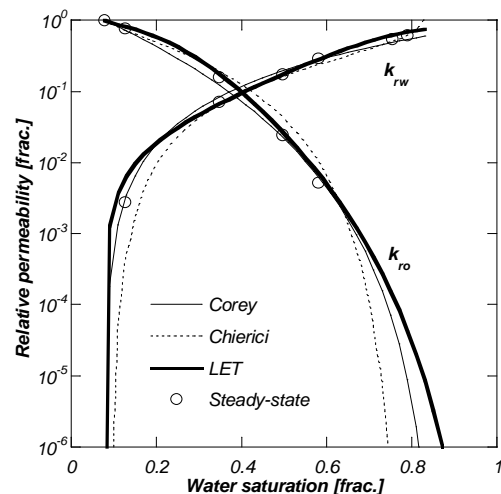


Figure 8: Relative permeability; semilog

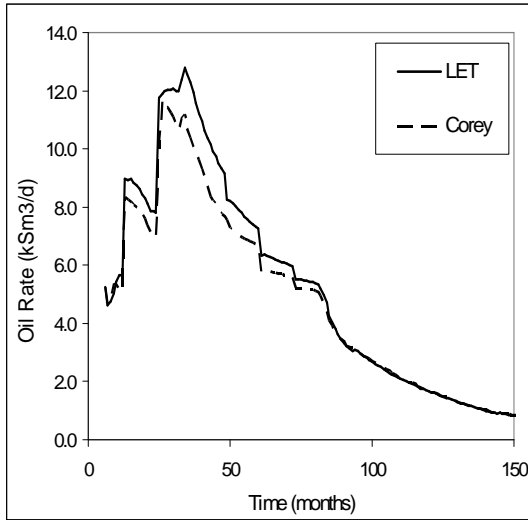
## FIELD APPLICATION

We will now demonstrate the use of the LET correlation in a field scale simulation. Oil from a field in the Norwegian Sea will be produced from a thin oil zone overlain by a gas cap and underlain by an active aquifer. The primary SCAL-program covered water-flooding and single-speed centrifuge experiments for determination of relative permeability curves. Reliable information of relative permeability is thus limited to high water saturations. Hence these data were initially represented by a simple Corey correlation that was used in the full field model. There was a clear mismatch between the full field simulations and the utilized well testing as the well test showed no water production while the simulations showed immediate water production.

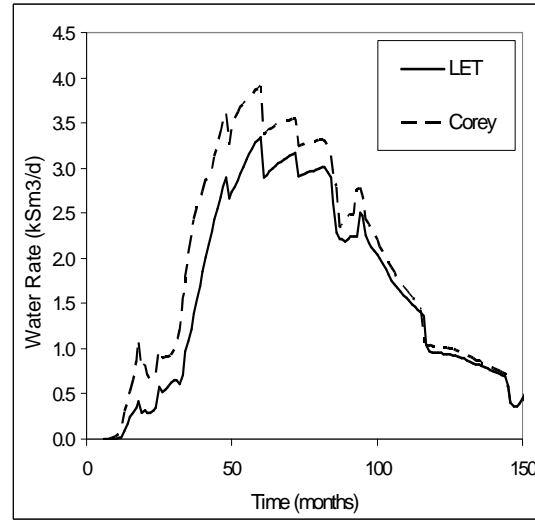
To history match the well test, an artificial value of the critical water saturation was introduced to delay the water breakthrough. This does not significantly influence the simulations, and the full field model still produced water too soon. The next remedy was to increase the oil mobility at low water saturations while still honouring the SCAL data at high water saturations. As the Corey correlation cannot increase one part of the relative permeability curve without affecting the entire curve, the LET correlation was introduced. The LET correlation was sufficiently flexible for this adjustment.

The results from the LET correlation for the oil phase were then implemented as the base case for the full field simulations. For comparison the full field model was also run with the results from the Corey correlation, and some results from a sector of the field are shown in figure 9 and 10. The relative permeability model influences the simulated field production almost from the start, and the effect lasts for a very long time as shown in figure 9 and 10. This is due to the rapid water breakthrough. Developing a field of this kind is a delicate balance of producing oil, water and gas. The gas breakthrough is just as rapid

as the water breakthrough. The gas production rates, however, are in general very similar for the two correlations (not shown).

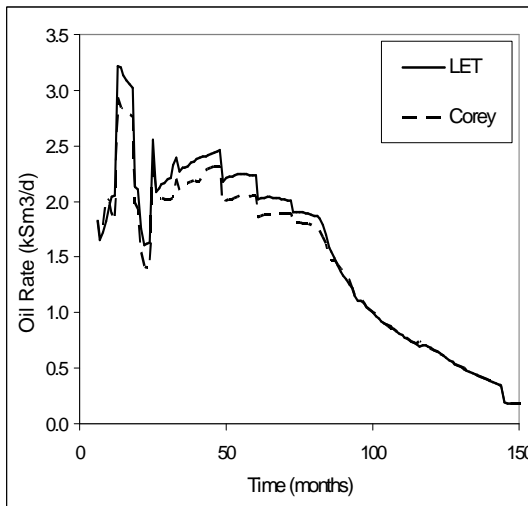


**Figure 9:** Oil production rate of a sector

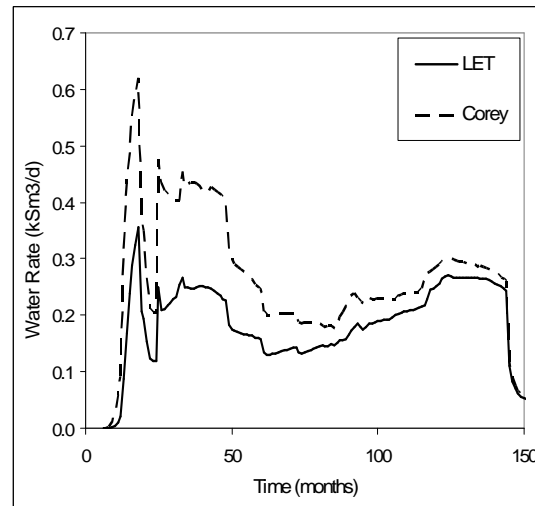


**Figure 10:** Water production rate of a sector

The simulated oil and water production of a single well is shown in figure 11 and 12. We note that the effect on the oil production is significant, but the effect on the water production is dramatic. In the first 4 years the simulated water production from the well using the Corey correlation is almost twice as large as the simulated water production from this well using the LET correlation.



**Figure 11:** Oil production rate of a well



**Figure 12:** Water production rate of a well

## CONCLUSIONS

- A new analytical relative permeability correlation has been developed; the LET correlation.
- Using only 3 parameters the correlation is able to describe a span of relative permeability curves and also reconcile differential pressure and production data through simulations of SCAL experiments.
- We have demonstrated that the use of 3 parameters allows control of the correlation over a broad range of saturations.
- Despite the addition of parameters, the LET correlation remains easily accessible and applicable for full field reservoir simulations and engineering.

## ACKNOWLEDGEMENT

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## NOMENCLATURE

$E_x^{yz}$	Empirical parameter for phase x with associated phase y and other information z	$k_{rw}^y$	Water relative permeability at residual saturation of phase y
$k_o(S_{wi})$	Oil permeability at initial water saturation [mD]	$L_x^{yz}$	Empirical parameter for phase x with associated phase y and other information z
$k_{rg}$	Relative permeability to gas	$S_{gn}$	Gas saturation, normalized
$k_{rg}^x$	Gas relative permeability at irreducible water saturation and zero oil saturation	$S_{grw}$	Residual gas saturation after water invasion
$k_{rg}^y$	Gas relative permeability at residual saturation of phase y	$S_{org}$	Residual oil saturation after gas injection
$k_{ro}$	Oil relative permeability	$S_{orw}$	Residual oil saturation after water injection
$k_{ro}^x$	Oil relative permeability at irreducible water saturation and zero gas saturation	$S_{wi}$	Irreducible water saturation
$k_{rog}$	Oil relative permeability with gas injection	$S_{wn}$	Water saturation, normalized
$k_{row}$	Oil relative permeability with water injection	$T_x^{yz}$	Empirical parameter for phase x with associated phase y and other information z
$k_{rw}$	Water relative permeability		

## APPENDIX A Water injection and oil production

We will now list the LET relative permeability correlations for water injection and oil production. Let us first define the normalized water saturation

$$S_{wn} = \frac{S_w - S_{wi}}{1 - S_{wi} - S_{orw}} \quad (A1)$$

The correlation for oil relative permeability with water injection is

$$k_{row} = k_{ro}^x \frac{(1 - S_{wn})^{L_o^w}}{(1 - S_{wn})^{L_o^w} + E_o^w S_{wn}^{T_o^w}} \quad (A2)$$

The correlation for water relative permeability with oil production is

$$k_{rw} = k_{rw}^o \frac{S_{wn}^{L_w^o}}{S_{wn}^{L_w^o} + E_w^o (1 - S_{wn})^{T_w^o}} \quad (A3)$$

The correlation for the endpoint of oil relative permeability is

$$k_{ro}^x = \frac{(1 - S_{wi})^{L_o^k}}{(1 - S_{wi})^{L_o^k} + E_o^k S_{wi}^{T_o^k}} \quad (A4)$$

The correlation for the endpoint of water relative permeability with oil production is

$$k_{rw}^o = \frac{(1 - S_{orw} - S_{wi})^{L_w^{ok}}}{(1 - S_{orw} - S_{wi})^{L_w^{ok}} + E_w^{ok} S_{orw}^{T_w^{ok}}} \quad (A5)$$

## APPENDIX B Gas injection and oil production

We will now list the LET relative permeability correlations for gas injection and oil production. Let us first define the normalized gas saturation

$$S_{gn} = \frac{S_g}{1 - S_{wi} - S_{org}} \quad (B1)$$

The correlation for oil relative permeability with gas injection is

$$k_{rog} = k_{ro}^x \frac{(1 - S_{gn})^{L_o^g}}{(1 - S_{gn})^{L_o^g} + E_o^g S_{gn}^{T_o^g}} \quad (B2)$$

The correlation for gas relative permeability with oil production is

$$k_{rg} = k_{rg}^o \frac{S_{gn}^{L_g^o}}{S_{gn}^{L_g^o} + E_g^o (1 - S_{gn})^{T_g^o}} \quad (B3)$$

The correlation for the endpoint of gas relative permeability with oil production is

$$k_{rg}^o = \frac{(1 - S_{org} - S_{wi})^{L_g^{ok}}}{(1 - S_{org} - S_{wi})^{L_g^{ok}} + E_g^{ok} S_{org}^{T_g^{ok}}} \quad (B4)$$

## APPENDIX C Water injection (or invasion) and gas production

We will now list the LET relative permeability correlations for water invasion and gas production. Let us first define the normalized water saturation

$$S_{wn} = \frac{S_w - S_{wi}}{1 - S_{wi} - S_{grw}} \quad (C1)$$

The correlation for gas relative permeability with water invasion is

$$k_{rgw} = k_{rg}^x \frac{(1 - S_{wn})^{L_g^w}}{(1 - S_{wn})^{L_g^w} + E_g^w S_{wn}^{T_g^w}} \quad (C2)$$

The correlation for water relative permeability with gas production is

$$k_{rw} = k_{rw}^g \frac{S_{wn}^{L_w^g}}{S_{wn}^{L_w^g} + E_w^g (1 - S_{wn})^{T_w^g}} \quad (C3)$$

The correlation for the endpoint of gas relative permeability is

$$k_{rg}^x = \frac{(1 - S_{wi})^{L_g^k}}{(1 - S_{wi})^{L_g^k} + E_g^k S_{wi}^{T_g^k}} \quad (C4)$$

The correlation for the endpoint of water relative permeability with gas production is

$$k_{rw}^g = \frac{(1 - S_{grw} - S_{wi})^{L_w^{gk}}}{(1 - S_{grw} - S_{wi})^{L_w^{gk}} + E_w^{gk} S_{grw}^{T_w^{gk}}} \quad (C5)$$